Calculus - Functions, an Introduction

Given 2 sets A and B:

$$A = \{0, 1, 2, 3\} \quad \text{and} \quad B = \{-1, 0, 1, 2, 3\}$$

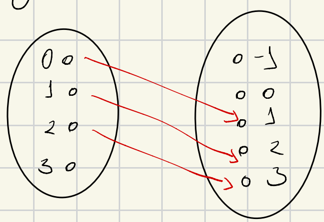
we have many binary possibilities, like:

$$R = \{(x,y) \in A \times B \mid y = x+1\}$$

or

$$S = \{(x,y) \in A \times B \mid y^2 = x^2\}$$

for each element $x \in A$, we have only one relation on $y \in B$ as: $(x,y) = R$



THIS ISN'T A FUNCTION RELATIONSHIP!

In other words, a proper function relation should be:

"FOR EVERY $x \in A$, there will be one, and only one, $y \in B$ as (x,y)."

So, the proper definition is:

f MEANS FUNCTION

$$A \times B \iff (\forall x \in A, \exists! y \in B \mid (x,y) \in f)$$

for each $y = x$, with $A = x$ and $B = y$, we say that A has image in B.

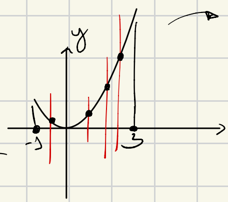
A is the <u>DOMAIN</u>
B is the <u>IMAGE</u>

<u>FUNCTION IN CARTESIAN PLANE</u>

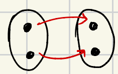
Is possible to visualize what is a function in a cartesian plane.

"(x,y) is ALWAYS ONE POINT IN TE PLANE"

One vertical line should pass in f, only one time.

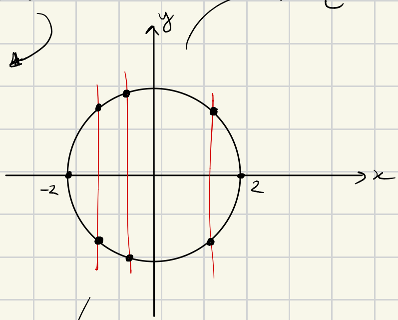


THIS <u>IS</u> A FUNCTION.
ONLY ON POINT BY VERTICAL LINE.

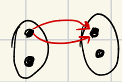


$A = \{x \in \mathbb{R} \mid -1 \leq x \leq 3\}$

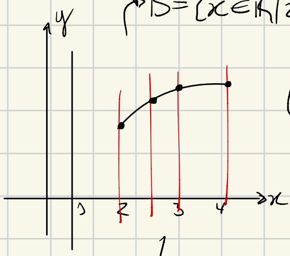
$A = \{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

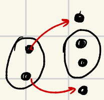


THIS IS <u>NOT</u> A FUNCTION.
THERE ARE MULTIPLE POINTS FOR THE SAME LINE



$B = \{x \in \mathbb{R} \mid 2 \leq x \leq 4\}$





THERE ARE NO IMAGES FOR y.
THIS IS <u>NOT</u> A FUNCTION.

A function can be defined with the following notations:

$$f: A \to B \quad | \quad A \xrightarrow{f} B \quad | \quad f: A \to B$$
$$x \mapsto f(x) \quad | \quad x \mapsto f(x) \quad | \quad y = f(x)$$

↳ THIS IS THE BETTER

following this notations, we can define better what is an image and what is a domain

$$f(a) = b$$

↳ b is the image of the value a applied in f

Using the same logic, a is the domain of the image.

$$\{\forall x \in A \mid y \in B \mid (x,y) \in f\}$$

if we call domain as D, is possible to point that:

$$x \in D \iff f(x) \in \mathbb{R}$$

Determining some domain by examples:

$y = 2x$
↳ $2x \in \mathbb{R}$ so, $\forall x \in \mathbb{R} \Rightarrow$ OUR DOMAIN IS $D = \mathbb{R}$

$y = x^2$
↳ $x^2 \in \mathbb{R}$ so, $\forall x \in \mathbb{R} \Rightarrow D = \mathbb{R}$

$y = \frac{1}{x}$

↳ $\frac{1}{x} \in \mathbb{R}$, so, $x \in \mathbb{R}$ only when $x \neq 0$.

that said, $\underline{D = \mathbb{R}^*}$

↳ $\mathbb{R}^*$ MEANS $\mathbb{R} - \{0\}$

$y = \sqrt{x}$

↳ $\sqrt{-x} = |x|$.

To $y \in \mathbb{R}$, $\sqrt{x} \in \mathbb{R}$ where $x \geq 0$.

$\underline{D = \mathbb{R}_+}$

$y = \sqrt[3]{x}$

↳ $\underbrace{\sqrt[3]{x} = x \text{ and } \sqrt[3]{-x} = -x}_{\underline{D = \mathbb{R}}}$

161. Dê o domínio das seguintes funções reais:

a) $f(x) = 3x + 2$	d) $p(x) = \sqrt{x - 1}$	g) $s(x) = \sqrt[3]{2x - 1}$
b) $g(x) = \frac{1}{x + 2}$	e) $q(x) = \frac{1}{\sqrt{x + 1}}$	h) $t(x) = \frac{1}{\sqrt[3]{2x + 3}}$
c) $h(x) = \frac{x - 1}{x^2 - 4}$	f) $r(x) = \frac{\sqrt{x + 2}}{x - 2}$	i) $u(x) = \frac{\sqrt[3]{x + 2}}{x - 3}$

a) $D = \mathbb{R}$, anything can multiply by 3 and sum 2.

b) $D = x < -2$ or $x > 0 \iff -2 < x > 0$

c) $D = \mathbb{R} - \{-2, 2\}$ d) $D = \mathbb{R}_+$ OR $\{x \in \mathbb{R} \mid x \geq 1\}$

↳ $\sqrt{x+1} \iff y = x + 1$, and $\sqrt{y}$

e) $D = \{x \in \mathbb{R} \mid x > -1\}$

f) $D = \{x \in \mathbb{R} \mid x \geq -2$ e $x \neq 2\}$

↳ if $x = -2$, then $\frac{0}{0} = 0$

g) $D = \mathbb{R}$

h) $\dfrac{1}{\sqrt[3]{2x+3}}$

$\hookrightarrow x = \dfrac{-3}{2} \Rightarrow 2\left(\dfrac{-3}{2}\right) = \dfrac{-6}{2} = -3$

$\Rightarrow \sqrt[3]{-3+3} = \sqrt[3]{0} = \text{No } \nexists$

$D = \{x \in \mathbb{R} \mid x \neq -\frac{3}{2}\}$

OR

$\mathbb{R} - \{-\frac{3}{2}\}$

i) $D = \mathbb{R} - \{3\}$

INDENTICAL FUNCTIONS

Two functions $f: A \to B$ and $g: C \to D$ are equals only if they have:

a) Equal domains $(A = C)$

b) Equal images $(B = D)$

c) $\forall x \mid f(x) = g(x)$