Properties for Limits that Tends to Infinity

a) $\pm\infty \pm \infty = \pm\infty$

b) $+\infty + k = +\infty$

c) $-\infty + k = -\infty$

d) $(+\infty)(+\infty) = +\infty$

e) $(-\infty)(-\infty) = -\infty$

f) $+\infty \cdot k = +\infty, \ k > 0$

g) $+\infty \cdot (-k) = -\infty, \ k < 0$

h) $\pm\infty \cdot 0 =$ Undefined

i) $k/0^+ = +\infty, \ k > 0$

j) $k/0^- = -\infty, \ k > 0$

k) $+\infty/0^+ = +\infty$

l) $+\infty/0^- = -\infty$

m) $\frac{\pm\infty}{\pm\infty} =$ Undefined

n) $0/0 =$ Undefined

o) $k/\pm\infty = 0$

Those are the functions behavior when tends to infinity

Some examples

When we have $\frac{k}{0}$, needs to check the lateral limits.

a) $\lim\limits_{x \to 5} \frac{x}{x-5} = \frac{5}{5-5} = \frac{5}{0} = \nexists$

$$\left.\begin{aligned} \lim_{x \to 5^+} \frac{x}{x-5} &= \frac{5}{0^+} = +\infty \\ \lim_{x \to 5^-} \frac{x}{x-5} &= \frac{5}{0^-} = -\infty \end{aligned}\right\} \lim_{x \to 5^+} \neq \lim_{x \to 5^-} \Rightarrow \lim_{x \to 5} = \nexists$$

b) $\lim\limits_{x \to 2} \frac{3x}{2x-4} = \frac{3(2)}{2(2)-4} = \frac{6}{4-4} = \frac{6}{0} = \nexists$

$$\left.\begin{aligned} \lim_{x \to 2^+} \frac{3x}{2x-4} &= \frac{6}{0^+} = +\infty \\ \lim_{x \to 2^-} \frac{3x}{2x-4} &= \frac{6}{0^-} = -\infty \end{aligned}\right\} \lim_{x \to 2^+} \neq \lim_{x \to 2^-} \Rightarrow \lim_{x \to 2} = \nexists$$

c) $\lim\limits_{x \to 2} \frac{x}{x^2-4} = \frac{2}{2^2-4} = \frac{2}{0} = \nexists$

$$\left.\begin{aligned} \lim_{x \to 2^+} &= \frac{2}{0^+} = +\infty \\ \lim_{x \to 2^-} &= \frac{2}{0} = -\infty \end{aligned}\right\} \lim_{x \to 2^+} \neq \lim_{x \to 2^-} \Rightarrow \lim_{x \to 2} = \nexists$$

d) $\lim\limits_{x \to 5^+} \frac{x}{x-3} = \frac{3}{3-3} = \frac{3}{0^+} = +\infty$

$\frac{k}{0^+} = +\infty$

e) $\lim\limits_{x \to 0} \sin(x) \cdot \operatorname{cosec}(x)$

$\lim\limits_{x \to 0} \sin(x) \longrightarrow$ If x tends to zero then $\sin(x)$ tends to zero.

$\sin(x) = 0$

$\lim\limits_{x \to 0} \operatorname{cosec}(x) = \frac{1}{\sin(x)} = \frac{1}{0}$

Since $\frac{1}{0} = \frac{k}{0}$, we need to check the laterals.

$\frac{1}{0^+} = +\infty \neq \frac{1}{0^-} = -\infty$

$\lim\limits_{x \to 0} \operatorname{cosec}(x) = \nexists$

$\lim\limits_{x \to 0} \sin(x) \operatorname{cosec}(x) = 0 \cdot \nexists = \nexists$