Limit is the behavior of a function when $x \to a$

↳ When x tends to a

$$\lim_{x \to 10} x^2 = 10^2 = 100$$

The following properties are true:

I) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

II) $\lim_{x \to a} [f(x) . g(x)] = \lim_{x \to a} f(x) . \lim_{x \to a} g(x)$

III) $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $g(x) \neq 0$

IV) $\lim_{x \to a} [c . f(x)] = \lim_{x \to a} c . \lim_{x \to a} f(x) = c . \lim_{x \to a} f(x)$

V) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

VI) $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x) \right]^n$

VII) $\lim_{x \to a} \ln [f(x)] = \ln \left[\lim_{x \to a} f(x) \right]$

VIII) $\lim_{x \to a} \text{sen/cos} [f(x)] = \text{sen/cos} \left[\lim_{x \to a} f(x) \right]$

IX) $\lim_{x \to a} e^{f(x)} = e \wedge \lim_{x \to a} f(x)$

Some examples

1. $\lim_{x \to \frac{1}{2}} 2x + 7 = \lim_{x \to \frac{1}{2}} 2x + 7$ → A constant is it's own limit

$$\lim_{x \to \frac{1}{2}} 2 . \frac{1}{2} = \frac{2}{2} = 1$$

$$\lim_{x \to \frac{1}{2}} 1 + 7 = 8$$

2. $\lim_{x \to -8} \sqrt[3]{x} = \sqrt[3]{\lim_{x \to -8} x} = \sqrt[3]{-8} = -2$

3. $\lim_{x \to 3} \frac{x^2 - 9}{x + 3} = \frac{\lim x^2 - \lim 9}{\lim x + \lim 3}$

$$= \frac{3^2 - 9}{3 + 3} = \frac{9 - 9}{6} = \frac{0}{6} = 0$$

4. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ → Without manipulation, this will lead to an undefined result.

$\frac{3^2 - 9}{3 - 3}$ can't be $\frac{0}{0}$, so $(x^2 - 9) = (x^2 - 3^2)$

and $(x^2 - 3^2) = (x - 3)(x + 3)$

$(x - 3)(x + 3) = x^2 + \cancel{3x} - \cancel{3x} - 9$

$= (x^2 - 9)$

$$\lim_{x \to 3} \frac{(\cancel{x - 3})(x + 3)}{\cancel{x - 3}} = \lim_{x \to 3} x + 3 = 6$$

5. $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$ → There's another way to manipulate

$4x^2 + 0x - 1 \mid 2x - 1$

$-4x^2 - 2x \quad\quad 2x + 1$

$0 \quad -2x - 1$

$2x + 1$

0

↳ This means $4x^2 - 1 = (2x + 1)(2x - 1)$

$$\lim_{x \to \frac{1}{2}} \frac{(2x + 1)(\cancel{2x - 1})}{\cancel{2x - 1}} = \lim_{x \to \frac{1}{2}} = 2x + 1 = \left(2 . \frac{1}{2}\right) + 1 = 2$$

6. $\lim_{x \to 0} x^2 \,\text{sen}\left(\frac{1}{x}\right) = \lim_{x \to 0} x^2 = 0$

$\lim_{x \to 0} \text{sen}\left(\frac{1}{x}\right) \to -1 \leq \text{sen} \leq 1$

$\lim_{x \to 0} -1 \leq \frac{1}{x} \leq 1 \quad [\times x^2]$

$-x^2 \leq x^2 \frac{1}{x} \leq x^2$ → Squeeze Theorem

$\lim_{x \to 0} -x^2 \leq x^2 \frac{1}{x} \leq x^2 \Rightarrow \lim_{x \to 0} x^2 = 0$

So, by the squeeze theorem: $\lim_{x \to 0} x^2 \,\text{sen}\left(\frac{1}{2}\right) = 0$