To find the A^{-1} of an $\underbrace{A_{n\times n} \text{ matrix}}_{\text{SQUARE MATRIX}}$ we do:

⚠ NOT EVERY MATRIX HAS AN INVERSE

Can be calc. by:

GIVEN $A_{n\times n}$, IT'S A^{-1} IS:

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

- if $\det(A)=0$, the matrix cannot have A^{-1}.
- if $\det(A)\neq 0$, go ahead.

To get the adj(A), first we need the COFACTORS MATRIX

FOR EACH a_{ij}

CALCULATES $C_{ij} = (-1)^{i+j} \cdot \det(M_{ij})$

↳ M_{ij} IS THE MATRIX GOT WHEN YOU REMOVE THE LINE i AND COLUMN j

And then $C^T = \operatorname{adj}(A)$.

So...

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

DETERMINANTS OF A MATRIX

I) MATRIX 1x1

$$\det[a] = a$$

II) MATRIX 2x2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc$$

$$\det\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = (3.2) - (4.1) = 6 - 4 = 2$$

III) MATRIX 3x3 (SARRUS)

$$A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(A) = aei + bfg + cdh - ceg - afh - bdi$$

$$\det(A) = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \text{OR} \quad \det(A) = \left[\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right]$$

$$\det\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad (1.5.9)+(2.6.7)+(4.8.3)-[(3.5.7)+(6.8.1)+(2.4.9)]$$

$$45+84+96-(105+48+72)$$

$$225-(225)$$

$$\det = 0$$

IV) MATRIX nxn (LAPLACE / COFACTORS EXPANSION)

Choose a line or column, prefer to use the one with most zeros.

$$\det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det(M_{ij})$$

↳ M_{ij} is get from removing the i (LINE) and j (Column) from A

$$A=\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 4 \\ 5 & -2 & 3 \end{bmatrix} \rightarrow \text{EXPANDING FROM THE FIRST LINE}$$

$$M_{1,1}\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} \quad \det(M_{1,1}) = 1.3-(-2.4) = 3+8 = 11$$

$$C_{1,1} = (-1)^{1+1}.11 = 11$$

$$A_{1,1}.C_{1,1} = 2.11 = 22$$

$$M_{1,2}\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix} \quad \det(M_{1,2}) = 3.3-5.4 = 9-20 = 11$$

$$C_{1,2} = (-1)^{1+2}.11 = -11$$

$$A_{1,2}.C_{1,2} = -1.11 = 11$$

$$M_{1,3}\begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} \det(M_{1,3})$$

$\hookrightarrow M_{1,3} = 0$

$$\det(A) = 22 + 11 + 0$$
$$= \underline{\underline{33}}$$

- The determinant of a triangular superior/inferior matrix is the product of the diagonal elements

$$\det\begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{bmatrix} = a \cdot e \cdot i$$

- The cofactor expansion is different from the cofactor formula.
  In this formula the term a_{ij} MUST be included and multiplied by its cofactor.
  This ensures the expansion accounts for the contribution of each element in the row / column.